Syllabus

I. Course Name: Geometry of submanifolds

II. Course description and objective

The course contains an introduction to differential topology, basic techiques and equations of the geometry of submanifolds, and some applications of these techniques.

III. Elective

IV. Bachelor Program, 8th Term, Hours, Credits

V. Course content

Section 1. Basics of topology and geometry of manifolds

Topic 1. Basic concepts of differential topology

Smooth manifold, smooth maps, tangent space, differential of map. Vector fields, Lie bracket, tensor fields, differential forms, submersions, immersions, embeddings, immersed and embedded submanifolds, Frobenius theorem, orientation, Sard theorem, embedding theorems, Whitney embedding theorem.

Topic 2. Induced metric and connection.

Isometric immersion, the first fundamental form of a submanifold, induced connection, the second fundamental form of a submanifold, Gauss decomposition, normal connection, Weingarten operator, Weingarten decomposition, mean curvature vector field. Examples and calculations.

Topic 3. Gauss-Codazzi-Ricci equations.

Gauss equation in the general case and in cases of a hypersurface, surface, submanifold in the spaces of constant curvature; the Codazzi equation in general and in some partial cases; Ricci's equation in general and in some partial cases; vector bundles; fundamental theorem for submanifolds in spaces of constant curvature.

Section 2. Classification of submanifolds and submanifolds in Euclidean space.

Topic 1. Basic classes of submanifolds

Totally geodesic submanifolds; totally umbilic submanifolds; axioms of planes and spheres by E. Cartan

Topic 2. Minimal submanifolds and submanifolds with a parallel mean curvature vector field.

Minimal submanifold – definition and examples; first and second variations of volume; stability of minimal submanifold, criteria, Jacobi fields; submanifolds with parallel mean curvature vector; surfaces of constant curvature, stability.

Topic 3. Hypersurfaces in the Euclidean space.

Totally umbilic hypersurfaces; Einstein hypersurfaces; convex hypersurfaces.

VI. Pre-taken courses: Topology, Differential Geometry, Riemannian Geometry

VII. Form of the final test: examination (four-level evaluation scale)/test (two-level evaluation scale)

VIII. Teaching materials and reference books

1. M. Dajczer. Submanifolds and isometric immersions. – Houston: Publish or Perish, 1990.
2. B.-Y. Chen. Geometry of submanifolds and its applications. – Tokyo, 1981.
3. T.H. Colding, W.P. Minicozzi II. Minimal surfaces. – NY: Courant Institute, 1999.
4. K. Kenmotsu. Surfaces with constant mean curvature. – Providence: AMS, 2003.

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