Syllabus

I. Course Name: Dynamical Systems

II. Course description and objective

The course is devoted to the contemporary theory of dynamical systems. We give an introduction to the general theory of dynamical systems, including concepts of limit sets, attractors, their fractal dimension, and quasi-stability method and consider examples of differential equations and study different approaches to the investigation of qualitative behavior of their solutions.

III. Elective

IV. Bachelor Program, 5th Term, 64 Hours, 4 Credits

V. Course content

Section 1. Basic concepts.

Discrete and continuous dynamical systems. Examples. Topological equivalence of dynamical systems.

Section 2. Trajectories and invariant sets.

Trajectories and semitrajectories, orbits, equilibria. Forward invariant, backward invariant, invariant sets, omega-limit and alpha-limit sets. Limiting properties of individual trajectories: Lagrange stability, Poisson stability. Recurrent properties of trajectories: wandering and non-wandering points, center of attraction, almost recurrent and recurrent trajectories, almost periodic trajectories.

Section 3. Lyapunov stability.

Section 4. 1D continuous systems.

1D systems with continuous reversible time. Comparison principle for 1D ODE. Lyapunov exponent.

Section 5. 2D continuous systems.

Poincaré-Bendixson theory. Examples.

Section 6. Bifurcation theory.

Pitchfork bifurcation. Transcritical bifurcation. Saddle-node bifurcation. Andronov-Hopf bifurcation. Saddle-node homoclinic bifurcation.

Section 7. Dissipative dynamical systems.

Criteria of dissipativity, examples.

Section 8. Asymptotic smoothness and compactness

Kuratowski measure of noncompactness. Equivalence of asymptotic smoothness and asymptotic compactness. Criteria of asymptotic compactness.

Section 9. Global attractors.

Existence of attractors. Stability properties and reduction principle. Stability with respect to parameters.

Section 2. Gradient systems.

Gradient systems. Lyapunov functions. Geometric structure of attractors. Criteria of existence of attractors for gradient systems.

Section 3. Dimension of attractors.

Fractal and Hausdorff dimensions. Criteria for finite dimensionality of invariant sets. Quasi-stable systems.

VI. Pre-taken courses

Mathematical Analysis, Measure Theory and Integration, Differential Equations

VII. Form of the final test: examination (four-level evaluation scale)/test (two-level evaluation scale)

VIII. Teaching materials and reference books

1. Chueshov I., Dynamics of quasi-stable dissipative systems, New York, Heidelberg, Berlin etc. : Springer, 2015.
2. Chueshov I., Introduction to the Theory of Infinite Dimensional Dissipative Systems, Kharkiv: Acta, 2002.
3. Chueshov I, Fastovska, T., Ryzhkova I., Quasistability method in study of asymptotical behaviour of dynamical systems J. Math. Phys. Anal. Geom. Vol. 15, no. 4, pp. 448–501, https://doi.org/10.15407/mag15.04.448.

Written by: Tamara Fastovska