Syllabus

1. Course Name: Functional Analysis II. **Instructor**: Volodymyr Kadets.

2. Course description and objective

The language of Functional Analysis is widely used in pure and applied mathematics. The course of advanced Functional Analysis enables students to understand better both the language itself and the ways to use this language in problems that come from other branches of mathematics, in particular from Harmonic Analysis. At the beginning, we recall without proofs the concepts and facts known from the basic Functional Analysis course. Afterwards, we study the new material: bases in Banach spaces, compactness criteria in infinite-dimensional spaces, spectral theory of compact operators in Banach spaces, selfadjoint operators in Hilbert spaces, including the machinery of functions of operators.

3. Compulsory. Master program, 1st semester, 64 hours, 5 credits.

4. Course content

Chapter 1. Basic principles of Functional Analysis.

In this chapter, we mainly recall with brief explanations the terminology and statements of most classical theorems from the introductory Functional Analysis course. More attention is paid to some important examples that usually are not covered by an introductory course.

Section 1. *Banach spaces. Linear operators. Norm of operator. Convergence in norm and pointwise convergence. Banach-Steinhaus theorem. Linear functionals. Dual space: basic examples. Hahn-Banach extension theorem. Extension by continuity. Projections and extension of operators*.

Section 2. *Quotient space, quotient map, injectivization of operator. Bounded below operators. Injectivity, surjectivity, invertibility. Isometry and isomorphism. Banach's inverse operator theorem. Closed graph theorem*.

Section 3. *Adjoint operator. Duality between operator and its adjoint. Duality between subspaces and quotient spaces*.

Chapter 2. Schauder bases and their applications.

In this chapter, we study the concept of basis in infinite-dimensional Banach space.

Section 1. *Convergent series. Definition of basis. Examples. Linear independence and completeness. Separability of the space and existence of basis. Coordinate functionals and partial sums operators. Pointwise convergence in spaces with bases. Criterion of basis.*

Section 2. *General form of linear functional in a space with basis. Examples. Bidual space. Reflexivity.*

Section 3. *Compact sets in a space with basis. Non-compactness of the unit ball. Examples.*

Chapter 3. Compact operators.

In this chapter, we study the concept of compact operator in infinite-dimensional Banach space, the concept of spectrum and demonstrate the spectral theorem for compact operators.

Section 1. *Finite rank operators and compact operators: basic properties and examples. Compactness of adjoint operator.*

Section 2. *Small perturbations of invertible operator. Spectrum and eigenvalues. Properties of spectrum. Resolvent and non-emptyness of the spectrum.*

Section 3. *Operators of the form "identity minus compact operator". Spectral theorem for compact operators.*

Chapter 4. Self-adjoint and unitary operators in Hilbert spaces.

In this chapter, we first recall the basic concept of Hilbert space theory, then pass to self-adjoint operators and their spectral properties, and afterward address the theory of functions of operators.

Section 1. *Hilbert spaces, Cauchy-Schwarz inequality, orthoprojectors, orthonormal bases. General form of linear functional.*

Section 2. *Bilinear forms and adjoint operators. Self-adjoint operators. Positive operators. Spectrum of self-adjoint operator. Polynomials in operator. Continuous functions in self-adjoint operators. Unitary operators. Polar representation.*

Section 3. *Borel functions in self-adjoint operators and spectral measure.*

5. Pre-taken courses (courses that students need to take before this course)

Mathematical Analysis, Linear Algebra, Real Analysis, Functional Analysis

6. Teaching methods

Lectures, solving problems in class and discussion

7. Form of the final test: examination (written part + oral part; four-level evaluation scale)

8. Teaching materials and reference books

V. Kadets. A course in functional analysis and measure theory // Springer, 2018, 539 pp.
https://www.springer.com/us/book/9783319920030

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