Syllabus

1. Course Name: Additional Topics in Functional Analysis. **Instructor**: Volodymyr Kadets.

2. Course description and objective

The course includes material for which there is no time in the basic course of Functional Analysis: theorems about fixed points and their applications, elements of topological groups, elements of Banach algebras, etc. The contents may variate on the request of the course participants.

3. Elective. Bachelor program, 7^{th} or 8^{th} semester, 64 hours, 4 credits.

4. Sample course content

Chapter 1. Fixed point theorems and applications.
Contractive mappings. Hausdorff distance. The existence of self-similar sets. The fixed point property. Brouwer's theorem. Partitions of unity and approximation of continuous mappings by finite-dimensional mappings. The Schauder's principle. The Picard and Peano theorems on the existence of a solution to the Cauchy problem for differential equations. The Lomonosov invariant subspace theorem. Kakutani's theorem on common fixed points of a family of mappings. Banach spaces with normal structure.

Chapter 2. Topological groups and Haar measure.
Definition and examples of topological groups. Properties of neighborhoods. Compact topological groups. Uniform continuity. Equicontinuity. Compactness criteria in C(G). Haar measure. Measure induced by a compact group of isomeries.

Chapter 3. Elements of Banach algebras.
Axiomatics and examples. Subalgebra. Homomorphism. Isomorphism. Invertibility in Banach algebras. The inversion formula. The spectrum. Liouville's theorem for Banach space-valued functions. The resolvent and non-emptiness of the spectrum. Ideals and quotient algebras. Commutative Banach algebras. Maximal ideals, complex homomorphisms and invertibility. Application: Wiener's theorem on absolutely convergent Fourier series. Gelfand compact, Gelfand homomorphism. Extreme points of convex sets, Krein-Milman theorem. The Stone-Weierstrass theorem. Gelfand-Neimark description of commutative C*-algebras.

5. Pre-taken courses (courses that students need to take before this course)

Mathematical Analysis, Linear Algebra, Measure and Integration Theory, General Topology, Functional Analysis.

6. Teaching methods

Lectures, solving problems in class and discussion

7. Form of the final test: examination (written part + oral part; four-level evaluation scale)

8. Teaching materials and reference books

•V. Kadets. A course in functional analysis and measure theory // Springer, 2018, 539 pp. https://www.springer.com/us/book/9783319920030
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•W. Rudin. Functional analysis // International Series in Pure and Applied Mathematics. New York, NY: McGraw-Hill. xviii, 424 p. (1991).

Written by Volodymyr Kadets