Syllabus

1. Course Name: Measure Theory and Integration. **Instructor**: Volodymyr Kadets.

2. Course description and objective

The course is devoted to the abstract Lebesgue integration theory and its application to functions of real variable. At the beginning we briefly touch metric and pseudometric spaces, and after that pass to the main subjects like measure spaces, extension of measures, Borel sets, measurable functions and integral, charges, Radon-Nikodym theorem, connection between integration and differentiation for functions of real variable.

3. Compulsory. Bachelor program, 5th semester, 64 hours, 4 credits.

4. Course content

Chapter 1. Metric and pseudometric spaces.
The axioms of pseudometric and metric. Sequences and topology. Lipschitz condition. Distance of a point to a set. Continuity of distance. Completeness. Nested sets theorem. Sets of first category and Baire's theorem. Cantor set. Compact sets in metric spaces. Compactness criteria in C(K).

Chapter 2. Measure theory.
Algebras of sets. Sigma-algebras of sets. Borel sets. Measures: finite and countable additivity. Measure spaces. Completeness. Completion of a sigma-algebra with respect to a measure. Atoms, purely atomic and non-atomic measures. Outer measure. Extension of measures. The Lebesgue measure on the interval. The meaning of the term ``almost everywhere''. Lebesgue's theorem on the almost everywhere differentiability of monotone functions. Existence of sets that are not Lebesgue measurable. Distribution functions and the general form of a Borel measure on the interval. Sigma-finite measures and the Lebesgue measure on the real line.

Chapter 3. Measurable functions.
Measurability criterion. Elementary properties of measurable functions. The characteristic function of a set. Simple functions. Lebesgue approximation of a measurable function by simple functions. Almost everywhere convergence. Convergence in measure. Egorov's theorem.

Chapter 4. The Lebesgue integral.

Simple integrable functions and their properties. Lebesgue integrable functions and Lebesgue integral. The integral as a set function. An integrability criterion for measurable functions. Chebyshev's inequality. The uniform limit theorem. Fatou's lemma. Lebesgue's dominated convergence theorem. Levi's theorems on sequences and series. Products of measure spaces. Double integrals and Fubini's theorem. Tonelli (inverse Fubini) theorem. The relation between Lebesgue integral, Riemann integral and the improper integral on an interval. The integral with respect to a sigma-finite measure.

Chapter 5. Absolute continuity of measures and functions. The connection between derivative and integral.

Charges. The boundedness of charges theorem. Jordan decomposition. The Hahn decomposition theorem. Absolutely continuous measures and charges. The charge induced by a function. The Radon-Nikodym theorem. The integral of a derivative. The normed space $L_1[a, b]$. The derivative of an integral as a function of the upper integration limit. Functions of bounded variation. Absolutely continuous functions. Recovering a function from its derivative.

5. Pre-taken courses (courses that students need to take before this course)

Mathematical Analysis, Linear Algebra

6. Teaching methods

Lectures, solving problems in class and discussion

7. Form of the final test: examination (written part + oral part; four-level evaluation scale)

8. Teaching materials and reference books

V. Kadets. A course in functional analysis and measure theory // Springer, 2018, 539 pp.
https://www.springer.com/us/book/9783319920030

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