Syllabus

I. Course Name: Partial Differential Equations II

II. Course description and objective

The area of Partial Differential Equations is not one unified topic, but rather a union of numerous direction of research with different sources of problems, motivations, and goals. The course focuses on the three most important aspects of the subject: (i) theory, which is to say the questions of existence, uniqueness, and continuous dependence on given data or parameters; (ii) phenomenology, meaning the study of properties of solutions of the PDEs that we examine; and (iii) applications, by which we mean that we will touch upon problems based on physical or geometrical applications. The topics we are going to discuss includes the application of the semigroup theory, calculus of variations, non-variational methods related to fixed point theorems.

III. Elective

IV. Master Program, 2nd Term, 64 Hours, 4 Credits

V. Course content

Chapter 1. Semigroup theory for Partial Differential Equations

Section 1. Basic ideas of the semigroup theory.

Elementary properties. Differential properties. Resolvent and its properties.

Section 2. Generating contraction semigroup.

Hille-Yosida Theorem.

Section 3. Applications of the semigroup theory to parabolic equations of the 2^{nd} order.

Parabolic equation as a contraction semigroup.

Section 4. Applications of the semigroup theory to hyperbolic equations of the 2^{nd} order.

Hyperbolic equation as a contraction semigroup.

Chapter 2. Variational methods for studying Partial Differential Equations

Section 5. Basic ideas of variational methods.

First variation. Euler-Lagrange equation. Second variation.

Section 6. *Euler-Lagrange equations for systems of equations*.

Null Lagrangians. Determinants as null Lagrangians.

Section 7. Minimizers.

Existence of minimizers. Coercivity. Lower semicontinuity. Convexity.

Section 8. Weak solutions of Euler-Lagrange equation.

Motivations to consider weak solutions of Euler-Lagrange equation. Systems of equations.

Section 9. Regularity of minimizers.

Estimates for the second derivative. The second derivative of minimizers. Higher regularity.

Section 10. Problems with constrains.

Nonlinear eigenvalue problems. One-sided constrains. Harmonic maps.

Chapter 3. Non-variational methods for studying Partial Differential Equations.

Section 11. Monotonicity methods.

Energy estimates. Existence and uniquness of weak solution.

Section 12. Fixed poin methods.

Strict contractions. Application of Banach's Fixed Point Theorem. Application of Schauder's and Schaefer's Fixed Point Theorems.

Section 13. Nonexistence of solutions.

Examples.

VI. Pre-taken courses

Mathematical Analysis, Measure Theory and Integration, Functional Analysis, Partial Differential Equations I.

VII. Form of the final test: examination (four-level evaluation scale)

VIII. Teaching materials and reference books

1. Evans L.C. Partial Differential Equations (2^{nd} ed.), American Mathematical Society, Providence, Rhode Island, 2010.
2. Craig W., A Course on Partial Differential Equations, Graduate Studies in Mathematics, 197, American Mathematical Society, 2018.

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