Syllabus

1. Course Name: Introduction to Banach space theory. **Instructor**: Volodymyr Kadets.

2. Course description and objective

The course of Functional analysis deals mostly with those theorems that work the same way in all Banach spaces. In contrast, in this course we mainly address those properties that distinguish one space from another.

The course covers the following subjects: block-bases technique in demonstration of pairwise non-isomorphism of spaces l_p. Khinchin's inequality, type and cotype of spaces L_p. Complementability, examples of non-complemented subspaces. Pełczyński's decomposition method in the proof that l_∞ and $L_\infty[0,1]$ are mutually isomorphic. Shrinking and boundedly complete bases, reflexivity criterion in terms of basis. Quasireflexive James space. Unconditionally convergent series and unconditional bases. A special role of c_o and l_1. Daugavet's theorem and the absence of unconditional bases in C[0,1] and $L_1[0,1]$. Finite representability. C-convexity and cotype, B-convexity and type.

3. Elective. Master program, 2nd or 3rd semester, 64 hours, 5 credits.

4. Course content

Chapter 1. Brief survey of duality and reflexivity in Banach spaces.
Bidual space, reflexivity. Weak convergence and its properties. Weak convergence criteria in classical spaces.

Chapter 2. Series in Banach spaces.
Convergence, absolute convergence and unconditional convergence. Unconditional convergence criteria. The Dvoretsky-Rogers theorem.

Chapter 3. Bases and basic sequences.
Separable and non-separable spaces. Complete and representing systems. Linear independence, minimality, and bi-orthogonal functionals. Markushevich bases. Schauder basis criterion. Basic constant, renorming that makes the basis monotone. Masur's theorem on basic sequence

existence. Equivalent sequences. Krein-Milman-Rutman theorem on small perturbations of a basis.

Chapter 4. Isomorphic classification.

Block-basis technique. Pairwise non-isomorphism of spaces l_p. Khinchin's inequality, type and cotype of spaces L_p. Isomorphic classification of spaces L_p. Complementability, examples of non-complemented subspaces. Pełczyński's decomposition method in the proof that l_∞ and $L_\infty[0,1]$ are mutually isomorphic.

Chapter 5. James theory.

Shrinking and boundedly complete bases, reflexivity criterion in terms of basis. Quasireflexive James space. Unconditional bases. A special role of c_o and l_1.

Chapter 6. Special properties.

Schur theorem on weak convergence in l_1. Quotient universality of l_1. Universality of C[0, 1]. Daugavet's theorem and the absence of unconditional bases in C[0,1] and $L_1[0,1]$. Weak unconditional convergence and Bessaga-Pełczyński c_o theorem. Daugavet's theorem and the absence of unconditional bases in C[0,1] and $L_1[0,1]$. Finite representability. C-convexity and cotype, B-convexity and type.

5. Pre-taken courses (courses that students need to take before this course)

Functional Analysis, Functional Analysis II.

6. Teaching methods

Lectures, solving problems in class and discussion

7. Form of the final test: examination (written part + oral part; four-level evaluation scale)

8. Teaching materials and reference books

•Albiac, Fernando; Kalton, Nigel J. Topics in Banach space theory. 2nd revised and updated edition. // Graduate Texts in Mathematics 233. Cham: Springer (ISBN 978-3-319-31555-3/hbk; 978-3-319-31557-7/ebook). xx, 508~p. (2016).

•Fabian, Marián; Habala, Petr; Hájek, Petr; Montesinos, Vicente; Zizler, Václav. Banach space theory. The basis for linear and nonlinear analysis. // CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC. Berlin: Springer (ISBN 978-1-4419-7514-0/hbk; 978-1-4419-7515-7/ebook). xiii, 820~p. (2011).

•Kadets, M. I.; Kadets, V. M. Series in Banach spaces: conditional and unconditional convergence. Transl. from the Russian by Andrei Iacob. // Operator Theory: Advances and Applications. 94. Basel: Birkhäuser. viii, 156 p. (1997).

•V. Kadets. A course in functional analysis and measure theory. Transl. from the Russian by Andrei Iacob. // Springer, 2018, 539 pp.

•Lindenstrauss, Joram; Tzafriri, Lior. Classical Banach spaces. 1: Sequence spaces. 2. Function spaces. Repr. of the 1977 and 1979 ed. // Classics in Mathematics. Berlin: Springer-Verlag. xx, 432 p. (1996).

Written by Volodymyr Kadets