Syllabus

I. Course Name: Differential Geometry II

II. Course description and objective

The course covers some additional topics in differential geometry and topology for undergraduates including basic Riemannian geometry, submanifolds, minimal surfaces, hyperbolic geometry, and de Rham cohomology.

III. Elective

IV. Bachelor Program, 6th Term, 120 Hours, 4 Credits

V. Course content

Section 1. Basic Riemannian geometry and submanifolds in Euclidean spaces

Topic 1. Tensor analysis and Riemannian geometry

Basics of smooth manifolds and maps, tangent spaces. Tensors, algebraic and differential operations on them. Vector fields. Riemannian metrics, examples. The length of a curve and angles between curves. Affine connection, curvature and Ricci tensors. Covariant derivative and parallel translation. Sectional curvature. Geodesics. Gauss-Bonnet theorem and Gauss integral formula. Vector analysis on manifolds.

Topic 2. Submanifolds in Euclidean spaces

The first and second fundamental forms, Gauss and Weingarten decompositions of a submanifold in Euclidean space. Normal, principal, and mean curvatures.

Topic 3. Minimal surfaces

Variations of length and area. Minimal surfaces and their examples. Minimal surfaces and harmonic functions. Weierstrass representation. Bernstein theorem. Constant mean curvature surfaces and their examples. Alexandrov theorem.

Section 2. Additional topics

Topic 1. Hyperbolic geometry

Models of hyperbolic space. Hyperbolic trigonometry. Hyperbolic isometries. Surfaces of constant negative curvature. Elliptic geometry.

Topic 2. De Rham cohomology

Exterior forms and their derivatives. Integration and Stokes theorem. De Rham cohomologies and their homotopic invariance. Poincaré lemma. Cohomologies of spheres. De Rham theorem.

VI. Pre-taken courses: Topology, Linear Algebra, Differential Geometry

VII. Form of the final test: examination (four-level evaluation scale)/test (two-level evaluation scale)

VIII. Teaching materials and reference books

1. M. do Carmo. Differential Geometry of Curves and Surfaces. Dover, 2016.
2. J. Jost. Riemannian Geometry and Geometric Analysis. Springer, 2017.
3. W.P. Thurston. Three-dimensional Geometry and Topology. Princeton University Press, 1997.
4. T.H. Colding, W.P. Minicozzi II. Minimal Surfaces. Courant Institute, 1999.
5. K. Kenmotsu. Surfaces with Constant Mean Curvature. AMS, 2003.
6. R. Bott, L.W. Tu. Differential Forms in Algebraic Topology. Springer, 1982.

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