Syllabus

I. Course name: Lie Algebras

II. Course description and objective

The course is devoted to the theory of Lie algebras, including structure of complex semisimple Lie algebras and their representations.

III. Elective

IV. Bachelor Program, 8th term, 64 hours, 4 credits

V. Course content

Section 1. Basic concepts of Lie algebras.

Definition of a Lie algebra. Category of Lie algebras. Basic examples. Lie algebras and linear algebraic groups. Lie functor. Representation of Lie algebras. Representations and g-modules. Irreducibility and complete reducibility.

Section 2. Solvable, nilpotent, and semisimple Lie algebras.

Solvable and nilpotent Lie algebras. Lie's and Engel's theorems. Invariant bilinear forms. Killing form. Cartan's criterion for solvability. Semisimple Lie algebras: definition, decomposition into a direct sum of simple ideals, Cartan's criterion for solvability, examples. Complete reducibility of finite dimensional representations of semisimple Lie algebras. Derivations of semisimple Lie algebras. Jordan decomposition in complex semisimple Lie algebras.

Section 3. Structure of semisimple Lie algebras.

Complex finite dimensional sl(2)-modules. Cartan subalgebras. Root decomposition and its properties. Root systems.

Section 4. Classification of semisimple Lie algebras and their representations.

Classification of complex semisimple Lie algebras. Classification of finite dimensional irreducible representations of complex semisimple Lie algebras.

VI. Pre-taken courses

Elements of Algebra and Number Theory, Linear Algebra, Abstract Algebra

VII. Form of the final test: examination (four-level evaluation scale)/test (two-level evaluation scale)

VIII. Teaching materials and reference books

1. J. E. Humphreys, Introduction to Lie Algebras and Representation Theory, Springer, 1973.
2. A. L. Onishchik, E. B. Vinberg, Lie Groups and Algebraic Groups, Springer, 1990.

Written by Eugene Karolinsky