Syllabus

1. Course Name: Topological Vector Spaces. **Instructor**: Volodymyr Kadets.

2. Course description and objective

In many problems of Analysis appear types of convergence on linear spaces of functions which cannot be described as convergence with respect to a norm. Such are, for instance, the pointwise convergence and the convergence in measure, the weak and weak* convergence in Banach spaces. An adequate language for describing the such topologies and types of convergence is that of topological vector spaces.

The course includes the following material: filters and compactness, axiomatics of topological vector spaces, metrizable spaces, locally convex spaces, operators and functionals, duality and weak topologies.

3. Elective. Master program, 2nd semester, 64 hours, 5 credits.

4. Course content

Chapter 1. Basic information about topological vector spaces.

Section 1. *Supplementary material from topology:* filters and filter bases; limits, limit points, and comparison of filters; ultrafilters; compactness criteria in terms of filters; topology generated by a family of mappings; Tikhonov product; theorem on Tikhonov product of compacts.

Section 2. *Background material on topological vector spaces:* axiomatics and terminology; neighborhoods of zero; metrizability; completeness, bondedness, precompactness, compactness of subsets; linear operators; continuity conditions for linear functionals; Hahn-Banach separation theorem.

Section 3. *Locally convex spaces*: seminorms and topology; Hahn-Banach extension theorem; weak topologies; quotient spaces; finite-codimensional subspaces; Eidelheit's interpolation theorem. .

Chapter 2. Elements of duality theory.

Section 1. *Duality in locally convex spaces:* the general notion of duality; polars; the bipolar theorem; barelled spaces and uniform continuity principle; the adjoint operator and weak continuity; Alaoglu's theorem.

Section 2. *Duality in Banach spaces:* weak-star convergence; the second dual space; Goldstine's theorem; weak convergence in Banach spaces; Mazur's theorem; total and norming sets; metrizability conditions; the Eberlein-Smulian theorem; reflexive spaces.

Chapter 3. The Krein-Milman theorem and its applications

Section 1. *Extreme points of convex sets:* definitions and examples; the Krein-Milman theorem; weak integrals and the Krein-Milman theorem in integral form.

Section 2. *Applications*: the connection between the properties of the compact K and those of C(K); the Stone--Weierstrass theorem; Bernstein's description of completely monotone functions; Lyapunov's theorem on vector measures.

5. Pre-taken courses (courses that students need to take before this course)

Mathematical Analysis, Linear Algebra, Functional Analysis

6. Teaching methods

Lectures, solving problems in class and discussion

7. Form of the final test: examination (written part + oral part; four-level evaluation scale)

8. Teaching materials and reference books

- V. Kadets. A course in functional analysis and measure theory // Springer, 2018, 539 pp. https://www.springer.com/us/book/9783319920030
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- Kothe, Gottfried. Topological vector spaces I. Grundlehren der mathematischen Wissenschaften. 159. New York: Springer-Verlag.

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