Syllabus

I. Course Name: Differential geometry of manifolds

II. Course description and objective

The course covers differential topology, analysis on manifolds, Riemannian and metric geometry.

III. Compulsory

IV. Master Program, 1st Term, 150 Hours, 5 Credits

V. Course content

Section 1. Analysis on smooth manifolds

Topic 1. Basics of differential topology

Manifolds and smooth structures. Smooth maps. Diffeomorphisms. Tangent vectors and a tangent space of a smooth manifold. Differential and rank of smooth map. Critical points and Sard theorem. Submersions. Immersions and embeddings. Whitney theorem. Submanifolds. Structure of the preimage of a regular value. Tensors. Tangent bundle. Vector fields. Lie brackets. Integration of vector fields. Tensor fields

Topic 2. Calculus of exterior differential forms

Forms in manifolds, symmetric and exterior forms. Pullback. Differential of the exterior form. Orientation of a smooth manifold. Integration of forms on an oriented manifold and the Stokes theorem.

Section 2. Riemannian geometry of manifolds

Topic 1. Fundamentals of Riemannian and metric geometry

Riemannian metric, Riemannian manifold. First fundamental form. The length of a curve on a Riemannian manifold and its properties. The angle between curves. The volume form and volume of a domain. Isometry and local isometry. Conformal maps. Affine connection. Covariant differentiation of forms. Riemann connection (Levi-Civita). The Kozshul formula. The length functional on the topological space. Inner metric. The shortest curve. The inner metric generated by a Riemannian metric. Finsler metric. Completeness of the Riemannian manifold. Geodesics of affine connection. Geodesics in the Riemannian manifold. Lagrangian and variational problem on a manifold. Euler-Lagrange equations. Geodesics and the shortest curves. The length of a curve in a metric space. The Arzela-Ascoli theorem for curves. Sufficient conditions for line to be the shortest. The Hopf-Rinow-Kon-Vossen theorem. Geodesic completeness. The case of a Riemannian manifold.

Topic 2. Curvature of Riemannian metric

The curvature operator, Riemann tensor, sectional curvature. Ricci curvature and scalar curvature. Einstein manifolds. Spaces of constant curvature. The Shur theorem. The second variation of the length of a geodesic line. Myers Theorem. Jacobi fields and conjugate points. Jacobi fields on manifolds of a constant sectional curvature. Jacobi's theorem and the index form of the geodesic. Exponential map. Hadamard spaces and the Cartan-Hadamard theorem. Comparison theorems. Metric spaces of bounded curvature.

VI. Pre-taken courses: Topology, Linear Algebra, Mathematical Analysis, Differential Geometry.

VII. Form of the final test: examination (four-level evaluation scale)

VIII. Teaching materials and reference books

1. Jürgen Jost. Riemannian Geometry and Geometric Analysis. Springer, 2017
2. Loring W. Tu. An Introduction to Manifolds. Springer, 2011
3. D. Burago, Y. Burago, S.Ivanov. A Course in Metric Geometry. AMS, 2001

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