Research statement
Dr. Iryna Ryzhkova-Gerasymova

My research interests lies manly in the area of infinite-dimensional dynamical systems. I study interactive (nonlinear) PDEs, like aeroelastisity, hydroelastisity models, their well-posedness and asymptotical behaviour. Aims of my study are existence of compact limit regimes and there qualitative properties like smoothness of limit trajectories, finite-dimensionality and in some cases their structure. I also interested in stability/stabilization to fixed points of solutions to (nonlinear) PDEs. Recently I've begun to adopt the quasistability method from dynamical system theory to non-autonomous processes.

Thermoelastic von Karman plate in a potential gas flow

My Ph. D. thesis was devoted to a number of nonlinear PDE models describing a plate subjected to thermal dissipation interacting with a gas flow [6, 5]. In study of asymptotical behaviour of these systems the problem is that the domain occupied by gas is unbounded and overall system gas+plate is not dissipative in corresponding norms. The idea is to consider energy of gas component in a system of halfballs of increasing radius. I decompose the overall problem in stable and compact components. In my case, rotational inertia of the plate or thermal dissipation allow to propagate compactness from the plate component to the gas component (in corresponding local energy topology) directly for energy solutions and prove stabilization (with respect to local energy topology) of solutions to the entire system to the fixed points of the system. This idea was one of the two later used to study asymptotical behaviour of plate+gas system without regularising effect of thermal damping (I. Lasiecka, J. Webster).

Hydroelasticity models (in collaboration with I. Chueshov)

We considered several models describing interaction of (nonlinear) plate with linear viscose fluid, occupying bounded domain (some types of unbounded domains may be included) [3, 2, 1]. The question is whether viscose dissipation in fluid component is sufficient to stabilize the whole system. We do not assume any dissipation in plate component. The answer appears to be positive for several types of plate equations. In the case of purely transversal displacements of the plate we proved finite dimensionality and additional smoothness of limit regimes set, using quasistability method. Partial smoothing effect introduced by the fluid plays important role in these considerations.

Full von Karman plate interacting with fluid Full von Karman equation describes nonlinear oscillations of a plate both in transversal and in-plane directions. In the case of rotational inertia neglected uniqueness of energy solutions to this equation is a challenging open problem. Even for the problem of interaction of this plate equation with viscose fluid the regularizing effects of the fluid do not allow to gain enough smoothness and prove uniqueness of energy solution. This question is still open. However, some result on well-posedness and asymptotical dynamics are obtained for strong solutions [4]. Boot strap technics for improving smoothness plays key in this work. I plan to continue this work towards to proving uniqueness of energy solution.

Current projects

1. *2D in space general hyperbolic systems (joint with I. Kmit).* There are a very few results on 2D in space general hyperbolic systems. We pose an issue of existence of periodical in time solutions to 2D in space hyperbolic systems of the first order, by now only for special cases of spacial domains. We intend to combine power of a-priori estimates and Fredholm theory to break the dimension barrier and gain insight what to do for general spacial domains.

2. *Numerical investigation of attractors of ODE (joint with A. Danilin).* Numerical errors can lead to completely wrong ideas on behavior of individual trajectory of a dynamical system.

Therefor in study of limit regimes the more useful idea is to simulate a bundle of trajectories for a (relatively) short time (see Dellnitz etc.) We work on a method of modelling Milnor's attractor of dissipative dynamical system. It is based on numerical simulation of bundle of trajectories, emanating from some set, and locating regions with high density of trajectories. By now this method is heuristical, but we work on rigorous justification. Some examples are available in the preprint `https://arxiv.org/abs/1808.04704`

3. *Finite dimensionality of attractors of non-autonomous processes.* I. Lasiecka and I. Chueshov designed a powerfull method (quasistability method) to prove finite dimensionality of attractors for a wide range of dynamical systems, both hyperbolic and parabolic. This method is based on decomposition of the difference of two trajectories form attractor into stable and compact parts. Now I work on generalization of this idea to non-autonomous processes.

References

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