

Syllabus

I. Course Name: Fourier Series and Fourier Transform

II. Course description and objective

The course is devoted to the theory of abstract Fourier series in arbitrary Euclidean space, trigonometrical Fourier series and some questions connected with Fourier transform and its analytical properties. A number of theorems on convergence in several senses and concrete examples are given.

III. Elective

IV. Bachelor Program, 5th Term, 64 Class Hours, 5 Credits

V. Course content

Chapter 1. Fourier series. Theorems on approximations of continuous functions.

Section 1. Some elementary functions in complex domain.

Section 2. Orthonormal systems of functions.

Section 3. Fourier series (FS) on the orthonormal system of functions.

Section 4. FS on the orthonormal system in Euclidean space. Extremal property of Fourier coefficients.

Section 5. Geometrical sense of extremal property of Fourier coefficients.

Section 6. Bessel inequality and Parseval equality.

Section 7. Basis in Euclidean space.

Section 8. FS on the orthogonal system. Trigonometrical Fourier Series (TFS). Bessel inequality and Parseval- Lyapunov equality.

Section 9. TFS for even and noneven functions.

Section 10. Expansion in TFS of periodic functions and on the arbitrary interval.

Section 11. Dirichlet integral.

Section 12. Riemann lemma.

Section 13. Principle localisation of Riemann.

Section 14. The main global criterion of termwise convergence of TFS.

Section 15. Dini criterion of termwise convergence of TFS, corollaries.

Section 16. Uniform convergence of TFS.

Section 17. Chesaro method. Cauchy theorem. Sum of TFS by Chesaro.

Section 18. Feyer theorem.

Section 19. Weierstrass approximation theorems.

Section 20. Integration and differentiation of Fourier series.

Chapter 2. Fourier integral and transform.

Section 1. Complex form of Fourier series.

Section 2. Fourier transform, exmples.

Section 3. Trigonometrical form of Fourier integral, example.

Section 4. Properties of Fourier transform.

Section 5. Dini criterion of convergence of Fourier integral.

Section 6. Convolution of functions and its Fourier transform.

Section 7. Cauchy problem for the heat conduction equation.

VI. Pre-taken courses

Mathematical Analysis, Functional Analysis, Ordinary Differential equations

VII. Form of the final test: control works.

VIII. Teaching materials and reference books

1. Фихтенгольц Г. М. Курс дифференциального и интегрального исчисления, т. III. – М.: Наука, 1966.
2. Дороговцев А.Я. Математичний аналіз, ч. II. – К. : Либідь, 1994.
3. Кудрявцев Л. Д. Курс математического анализа, т. II. – М.: Высшая школа, 1988.
4. Никольский С. М. Курс математического анализа, т. II. – М.: Наука, 1973.
5. Кудрявцев Л. Д., Кутасов А. Д. и др. Сборник задач по математическому анализу. Интегралы. Ряды. – М.: Наука, 1986.
6. Демидович Б. П. Сборник задач и упражнений по математическому анализу. – М.: Наука, 1977.
7. Дороговцев А. Я. Математичний аналіз (Збірник задач). – К.: Вища школа, 1987.

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