Syllabus

I. Course name: Algebra II

II. Course description and objective

The course is devoted to several topics in algebra that are very important but not normally covered by undergraduate courses. First, we discuss and/or recall definitions and examples of basic algebraic notions. Further, we consider language of categories, tensor products, basic commutative algebra (including modules over principal ideal domains), and basic representation theory. If time allows, other topics can be considered.

III. Compulsory

IV. Master Program, 1st term, 64 hours, 5 credits

V. Course content

Section 1. Basic algebraic structures.

Groups. Rings and fields. Modules and vector spaces.

Section 2. Language of categories and functors.

Categories: definition and examples. Universal objects. Functors. Morphisms of functors. Equivalence of categories.

Section 3. Tensor products and multilinear algebra.

Tensor product of modules over a commutative ring: definition, existence, and uniqueness. Canonical isomorphisms. Base change (extension and restriction of scalars). Tensors in coordinates. Symmetric and skew-symmetric tensors. Multivectors and Grassmannians. Plücker embedding.

Section 4. Modules over principal ideal domains.

Euclidean domains, principal ideal domains, and unique factorization domains. Free modules over principal ideal domains. Invariance of rank. Aligned bases and Smith normal form. Structure of finitely generated modules over principal ideal domains. Applications to finitely generated abelian groups. Application to linear operators: Frobenius and Jordan normal forms.

Section 5. Basic representation theory.

Associative algebras and their representations. Simplicity and indecomposability. Schur's lemma. Jacobson radical of finite dimensional associative algebra and semisimplicity. Characters. Jordan-Hölder and Krull-Schmidt theorems. Representations of finite groups.

VI. Pre-taken courses

Elements of Algebra and Number Theory, Linear Algebra, Abstract Algebra

VII. Form of the final test: examination (four-level evaluation scale)

VIII. Teaching materials and reference books

1. Hungerford, T. W., Algebra, Springer, 1980.
2. Vinberg, E. B., A Course in Algebra, American Mathematical Society, 2003.
3. Knapp, A. W., Basic Algebra, Springer, 2006.
4. Knapp, A. W., Advanced Algebra, Springer, 2007.
5. Gorodentsev, A. L., Algebra I, Springer, 2016.
6. Gorodentsev, A. L., Algebra II, Springer, 2017.
7. Rotman, J. J., Advanced Modern Algebra: Third Edition, Part 1, AMS, 2015.
8. Rotman, J. J., Advanced Modern Algebra: Third Edition, Part 2, AMS, 2017.

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