Syllabus

I. **Course Name**: Complex Analysis

II. **Course description and objective**

III. The main objectives of the discipline are to teach students the theoretical basics and methods of complex analysis and to apply these methods in other mathematical disciplines.

IV. **Compulsory**

V. **Bachelor Program, 5-6th Term, 180 Hours, 6 Credits**

VI. **Course content**

Chapter 1. Basic concepts of complex analysis.

Theme 1. Complex plane and functions of complex variable.

1. Complex numbers, actions with complex numbers.

2. Definitions of trigonometric functions of a complex variable.

3. Complex plane topology, stereographic projection, extanded complex plane.

4. Functions of a complex variable, domains and curves in the complex plane

Theme 2. Differentiation of functions, holomorphic and harmonic functions

1. R- and C- differentiation of functions of a complex variable.

2. Cauchy-Riemann conditions.

3. Definition of holomorphic function.

4. Geometric content of module and argument of holomorphic function.

5. Harmonic functions. Properties of harmonic functions.

6. Relationship between harmonic and holomorphic functions. Recovery of a holomorphic function by a given real part.

Theme 3. The integral of functions of a complex variable function and the Cauchy theorem.

1. Definition of the integral along the curve and its properties.

2. Relationship between curvilinear integrals.

3. Newton-Leibniz formula. Primitive.

4. The Cauchy theorem for the triangle.

5. The Cauchy theorem for a closed curve in a simply connected domain.

6. The Cauchy theorem for a function continuous in a closed domain.

Theme 4. The Cauchy integral formula and its application

1. The Cauchy integral formula.

2. Differentiation of the Cauchy type integral.

3. Infinite differentiation of holomorphic functions. Morerey theorem.

4. Weierstrass theorem on a uniformly convergent sequence of holomorphic functions.

5. Power series.

6. The Power-series expansion of holomorphic functions.

7. Cauchy inequality for coefficients of power series.

8. Liouville theorem.

Section 2. Zeroes, isolated singularities, residuals.

Theme 5. Zeros of holomorphic functions and direct analytical continuation.

1. Zeros of holomorphic functions

2. The first uniqueness theorem.

3. The theorem is that zeros cannot be condensed.

4. Direct analytical continuation.

5. Singularities of power series on the boundary of the circle of convergence.

Thime 6. The Laurent series and isolated singularities.

1. The Laurent series, the expansion of a holomorphic function in the Laurent series.

2. Various types of isolated singular points.

3. The Sokhotsky-Weierstrass theorem.

4. Residuals. Calculation of residuals.

5. The Cauchy theorem on residuals.

Theme 7. Application of the Cauchy theorem on residuals.

1. Calculation of integrals on a closed loop.

2. Lemma Jordan

3. Calculation of integrals from trigonometric functions.

4. Calculation of improper integrals.

5. Summation of series.

Section 3. Further properties of holomorphic functions.

Theme 8. Geometric principles of function theory.

1. The principle of the argument.

2. Rouchet and Hurwitz theorems.

3. Basic theorem of algebra.

4. The principle of conservation of domains.

5. One-sheeted functions.

6. Inversion of power series.

7. Principle of maximum modulus of holomorphic function.

8. Schwartz Lemma.

Theme 9. Properties of entire and meromorphic functions.

1. Definition of entire and meromorphic functions.

2. Rational functions.

3. The Mittag-Leffler expansion of meromorphic functions.

4. The Cauchy method of expansion of meromorphic functions.

5. The Weierstrass canonical factor.

6. Infinite product and its properties.

7. Weierstrass theorems on representation of an entire function as an infinite product.

Section 4. Conformal mappings and their applications.

Theme 10. Elementary conformal mappings.

1. The definition of a conformal mapping.

2. Necessary and sufficient conditions for conformality.

3. Linear-fractional mappings and their properties.

4. Functions z^n, $\sqrt[n]{z}$ and their properties.

5. Functions e^z, $\ln z$ and their properties.

6. Zhukovsky's function and its inverse.

7. Construction of conformal mappings of simple connected domains by using linear-fractional functions, z^n, $\sqrt[n]{z}$, e^z, $\ln z$, Zhukovsky's function.

Thime 11. The main theorem of the theory of conformal imagery.

1. Conformal automorphism and isomorphism.

2. The connection between conformal mappings and holomorphic functions.

3. Classes of conformally equivalent domains.

4. Painlevé's theorem on removal of singularities.

5. Riemann-Schwartz' principle of symmetry.

Theme 12. The Dirichlet problem and its application to the theory of conformal mappings

1. The Dirichlet problem for the circle. Poisson formula.

2. The Dirichlet problem for simply connected domains.

3. The Dirichlet problem for the upper halfplane

4. Christoffel-Schwartz' formula

VI. Pre-taken courses

Mathematical Analysis, Algebra

VII. Form of the final test: test (two-level evaluation scale) + examination (four-level evaluation scale)

VIII. Teaching materials and reference books

Basic Literature

1. B.V.Shabat. Introduction to complex analysis. V.1, M., "Science", 1985 (Russian).
2 E.C. Titchmarsh The Theory of Functions Oxford University Press 1939
3. M.A. Lavrentiev, B.V. Shabat. Methods of the theory of functions of a complex variable. M., "Science", 1973 (Russian).
4. A collection of problems on the theory of analytic functions (edited by M. A. Evgrafov) M .. "Science", 1972.(Russian).

Additional literature

1. L. Alfors. Complex analysis. N.J., "Kluver", 1981.

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