Syllabus

I. Course Name Navier-Stokes equations

II. Course description and objective We start with functional spaces we need to study solutions to Navier-Stokes equations. Then we consider linearized equations (both static and dynamic). We study nonlinear Navier-Stokes equations with emphasis on 3D equations and challenges related to the dimension. We also pay attention to numerical schemes of solution of the equations.

III. Elective

IV. Master Program, 3st Term, 32 Hours, 4 Credits

V. Course content

Section 1. Functional spaces. Divergence-free functional spaces. Normal trace. Weil decomposition.

Section 2. Stationary Stokes equations (linear). Weak solutions. Existence and uniqueness of solution. Non-homogeneous Stokes equations. Regularity. Eigenvectors of Stokes operator.

Section 3. Stationary Navier-Stokes equations. Weak solutions. Existence and uniqueness of solution. Compactness method. Some results on non-uniqueness.

Section 4. Evolution Navier-Stokes equations. Linear case: existence and uniqueness. 2D Nonlinear case: existence and uniqueness. 3D case: some results on existence, uniqueness and smoothness of solutions. Energy equality and inequality.

Section 5. Numerical schemes. Approximations of functional spaces of divergence-free functions. Consistent and non-consistent finite elements. Numerical schemes for stationary and evolution Navier-Stokes problems, their stability and convergence.

VI. Pre-taken courses Mathematical Analysis, Measure Theory and Integration, Functional Analysis, Sobolev spaces, Partial Differential Equations I.

VII. Form of the final test examination (four-level evaluation scale)/test (two-level evaluation scale).

VIII. Teaching materials and reference books

Temam R., Navier-Stokes Equations: Theory and Numerical Analysis (2nd ed.) North Holland, Amsterdam, 1984.

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