Exam Card sample

The Exam Card contains 8 problems of varying complexity that cover topics in Analysis, Linear Algebra, Geometry and Differential Equations.

Card 6

1. Solve the differential equation $\dot{x} + \frac{1}{t}x = 0$;
2. Find the general solution to the following system of differential equations:

$$\begin{cases} \dot{x}_1 = 2x_2, \\ \dot{x}_2 = -2x_1 + 1; \end{cases}$$

3. Analyze the function $f(x) = x - e^x$ by using the first and second derivatives and plot the graph;
4. Calculate $\int\limits_0^{\pi/4} x \cos 2x dx$;
5. Find eigenvalues and eigenvectors for the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 0 & 0 & 0 \end{pmatrix}$;
6. Reduce the quadratic form $Q(x_1, x_2) = x_1^2 - 3x_1x_2 + 2x_2^2$ to the diagonal form, find out if this form is positively definite;
7. Find the equation of a line perpendicular to $x - 4y - 7 = 0$ that passes through the center of the circle $x^2 + x + y^2 = 15$;
8. Find the values of parameter p for which there is a plane passing through the line $\frac{x-1}{p} = \frac{y+3}{-1} = \frac{z+2}{5}$ in parallel to the plane $4x + 3y - z + 3 = 0$.

Evaluation criteria: Complete and correct answer to each of Problems 1 – 7 has a score of 12 points; Problem 8 has a score of 16 points. If the solution to a problem requires clarification of some facts or tools, then it is required to give a background of the solution by giving a proof or reference to the corresponding theorem. Partial solution will be evaluated in accordance with the content.