

Syllabus

I. Course Name: Riemannian Geometry

II. Course description and objective

The course presents an introduction to the Riemannian geometry and its applications.

III. Elective

IV. Bachelor Program, 4th Term, 48 Hours, 4 Credits

V. Course content

1. *Elements of Differential Topology.* Smooth manifolds. Tangent and cotangent bundles. Tensor fields. Mappings of manifolds.

2. *Fundamental concepts of Riemannian Geometry.* Semi-Riemannian and Riemannian metric tensors. Lengths of curves. Angles between curves. Area/volume of domains. Isometric mappings

3. *Affine manifolds.* Covariant derivative. Parallel translation. Curvature and torsions. Geodesics.

4. *Levi-Civita connection.* Parallel translation on Riemannian manifolds.

5. *Geodesics on Riemannian manifolds.* Length and energy functionals. First variation formulas, extremality of geodesics. Exponential mapping.

6. *Completeness of Riemannian manifolds.* Hopf-Rinow theorem.

7. *Curvatures of Riemannian manifolds.* Riemannian curvature tensor, algebraic and differential properties. Sectional curvature, Ricci curvature, scalar curvature. Spaces of constant curvature. Schur theorem.

8. *Jacobi fields.* Second variation formula for length / energy of geodesics. Jacobi equation. Conjugate points of geodesics. Comparison theorems for Jacobi fields.

9. *Diameter and curvature.* Bonnet theorem. Myers theorem.

10. *Cartan-Hadamard theorem.*

11. *Injectivity radius.* Klingenberg theorem.

12. *Elements of General Relativity.*

VI. Pre-taken courses

Differential Geometry, Mathematical Analysis, Topology

VII. Form of the final test

Examination (four-level evaluation scale)

VIII. Teaching materials and reference books

1. Burago Y., Zalgaller V., *Introduction to Riemannian Geometry*. S.-P., Nauka, 1994.
2. Lee J.M. *Riemannian manifolds*. Springer, 1997.
3. Sternberg S., *Lectures on Differential Geometry*. AMS, 1999.
4. Berge M., *A panoramic view of Riemannian Geometry*. Springer, 2003.
5. Dubrovin B., Novikov S., Fomenko A., *Modern geometry. Methods and applications*. Nauka, 1986.

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