**Syllabus**

**I. Course Name**: Lie groups and homogeneous spaces

**II. Course description and objective**

The course introduces students to the Lie theory and its applications to differential geometry, including invariant geometry of homogeneous and symmetric spaces.

**III. Elective**

**IV. Master Program, 3rd Term, 120 Hours, 4 Credits**

**V. Course content**

**Section 1. Lie groups, Lie algebras, and invariant metrics**

**Topic 1. Lie groups, examples, basic properties**

Classical matrix groups. Topological groups, basic definitions, examples. Lie groups, basic definitions, examples. Left and right translations, the unit component and its properties. Topological structures of some classical groups. Tangent spaces of classical groups.

**Topic 2. Left invariant fields, exponential map, Lie bracket**

Left invariant fields and their properties. Exponential map and its properties. One-parameter subgroups. The Lie bracket construction. Lie brackets of left invariant fields.

**Topic 3. Lie algebras**

Algebras and associative algebras. Lie algebras, properties, examples. The Lie algebra of a Lie group.

**Topic 4. Representations of Lie groups and algebras**

Group representations. Representations of Lie groups, examples. The adjoint representation. Representations of Lie algebras, examples. The adjoint representation of a Lie algebra.

**Topic 5. Basic theorems of Lie theory**

Lie subgroups and subalgebras. Homomorphisms of Lie groups and Lie algebras. The existence and uniqueness of a Lie group with a given Lie algebra.

**Topic 6. Left invariant and biinvariant metrics**

Left invariant metrics, constructions, examples. Left invariant connections, explicit expressions for connections and curvatures of left invariant metrics. Biinvariant metrics, their connections, curvatures, and geodesics. Simple, semisimple, reductive, compact, solvable Lie algebras. The description of Lie groups that admit biinvariant metrics.

**Section 2. Homogeneous and symmetric spaces**

**Topic 1. Group actions on manifolds, homogeneous spaces**

Group actions and their properties. Smooth actions of a Lie group on a smooth manifold. Stabilizers and their properties. Examples. The smooth structure on a homogeneous space of a Lie group.

**Topic 2. Invariant Riemannian metrics**

Invariant metrics. Riemannian homogeneous spaces. Description of invariant metrics. Compactness of the stabilizer. Riemann submersions and O'Neill's formula. Curvatures of invariant metrics. Normal metrics. The Berger's example.

**Topic 3. Symmetric spaces**

Locally and globally symmetric spaces, definitions and examples. The algebraic description of a symmetric space. Irreducible spaces. Compact and non-compact type. Curvatures of symmetric spaces.

**VI. Pre-taken courses:** General Algebra, Linear Algebra, Topology, Differential Geometry of Manifolds

**VII. Form of the final test**: examination (four-level evaluation scale)

**VIII. Teaching materials and reference books**

1. S. Helgason. Differential Geometry, Lie Groups, and Symmetric Spaces. Providence: AMS, 2001.
2. R. Bryant. An introduction to Lie groups and symplectic geometry // In: Geometry and Quantum Field Theory. – Providence: AMS, 1995.
3. A. Besse. Einstein Manifolds. – Berlin: Springer-Verlag, 1987.

**Written by:** Eugene Petrov