

Syllabus

I. Course Name: Elements of Algebra and Number Theory

II. Course description and objective

This is an introductory course in Linear Algebra and General Algebra. The course covers the following topics: an elementary introduction to the number theory, complex numbers, polynomials, rational functions and permutations.

III. Compulsory

IV. Bachelor Program, 1st Term, 180 Hours, 6 Credits

V. Course content

Chapter 1. Introduction to Number Theory

1. Divisibility of integers, division with remainder.
2. Greatest common divisor, Euclidean algorithm.
3. Coprime integers, their properties.
4. Linear Diophantine equations.
5. Primes and the Fundamental Theorem of Arithmetic.
6. Congruence classes.
7. Sun Tzu's theorem (the Chinese remainder theorem).
8. Euler's φ -function and Euler's theorem.

Chapter 2. Complex numbers

1. Complex numbers. Addition and multiplication of complex numbers.
2. Trigonometric form of complex numbers. De Moivre's formula.
3. Roots of complex numbers. Roots of unity and their properties.
4. Cubic equations with complex coefficients.
5. Quartic equations with complex coefficients.
6. The definition of a field. Examples of fields.

Chapter 3. Polynomials in one variable over a field.

1. Polynomials in one variable over a field. Addition and multiplication of polynomials.
2. Degree of a polynomial. Division of polynomials with remainder.

3. Greatest common divisor of polynomials, Euclidean algorithm.
4. Coprime polynomials, their properties.
5. Irreducible polynomials. Factorization of polynomials.
6. Fundamental theorem of algebra and its corollaries.
7. Zeros of polynomials. Little Bézout's theorem. Multiple zeros.
8. Derivative of a polynomial. Connection between derivative and multiple zeros.
9. Taylor's formula.
10. Polynomial interpolation. The Lagrange and the Newton forms of the interpolation polynomials.
11. The definition of a ring. Examples of rings.

Chapter 4. Rational functions in one variable over a field.

1. Rational functions in one variable over a field: definition and properties.
2. The canonical form of a complex rational function in one variable.
3. The canonical form of a real rational function in one variable.

Chapter 5. Permutations.

1. The definition of a permutation. Composition of permutations, its properties.
2. Decomposition of permutations as products of disjoint cycles.
3. Decomposition of permutations as products of transpositions.
4. Inversions and the sign of a permutation.
5. The sign of a permutation represented as a product of transpositions.
6. The definition of a group. Examples of groups.

VI. Pre-taken courses

School Course of Mathematics

VII. Form of the final test: examination (four-level evaluation scale)

VIII. Teaching materials and reference books

1. Burton, David M. Elementary Number Theory. Allyn and Bacon, 1976. ISBN: 9780205048144.

2. Franz Lemmermeyer's lecture notes online
(<http://www.fen.bilkent.edu.tr/~franz/nth06/nth.pdf>)
3. A. G. Kurosh, Course of higher algebra, Publisher: MIR Publishers (1975)
Language: English ISBN-10: 0828507244, ISBN-13: 978-0828507240.
4. M. Bocher, Introduction to Higher Algebra, Dover Books on Mathematics,
2004, ISBN-10: 0486495701, ISBN-13: 978-0486495705.

Written by: Anna Vishnyakova