Syllabus

I. **Course Name**: Equations of mathematical physics

II. **Course description and objective**

The course is devoted to the theory of classical solutions to boundary, initial, and initial boundary value problems for classical partial differential equations.

III. **Compulsory**

IV. **Bachelor Program, 7-8th Term, 128 Class Hours, 8 Credits**

V. **Course content**

7th Term

Introduction. General theory of partial differential equations.

Basic notions. Classification of linear second-order partial differential equations.

Chapter 1 Elliptic equations.

Section 1. Boundary value problems for the Laplace equation. Classical solutions.

Section 2. Harmonic functions.

Green formulas for elliptic differential operators. Harmonic functions. Mean value theorems. Maximum principle for harmonic functions and its collorarіes.

Section 3. Green's function for the Laplace equation.
Green's function for the Dirichlet boundary value problem, its properties. Construction of Green's functions for 3D domains and 2D domains.

Section 4. Laplace equation in a ball.

Green's function for the Dirichlet boundary value problem in a ball. Classical solution to the Dirichlet boundary value problem for the Laplace equation in a ball.

Section 5. Fourier method for the Poisson equation in circlular domains.

Section 6. Potentials.

Volume potential and its properties. Single layer and double layer potentials and their properties. Gauss' lemma. Limiting properties at the boundary. Reduction of the Dirichlet and Neumann boundary value problems for the Laplace equation to integral equations. Theorems on the existence and uniqueness of classical solutions in nD and 2D cases.

Chapter 2. Special functions

Section 1. Cylindrical functions.

Cylindrical functions and their basic properties. Bessel, Hankel and Neumann functions. Recurrence relations. Asymptotics. Boundary value problems for the Bessel equation. Fourier method for the boundary value problems for the Laplace equation in cylindrical domans.

Section 2. Spherical functions.

Legendre polynomials and their generating function. Recurrence relations. Rodrigues' formula. Legendre's equation. Orthogonality and norms of the Legendre polynomials. Associated Legendre polynomials and their properties. Fourier method for the boundary value problems for the Laplace equation in spherical domains.

8th Term

Chapter 3. Parabolic equations

Section 1. Initial value problem for the heat conduction equation.

Classical solutions to the initial value problem for the heat conduction equation. *Maximum principle and its corollaries. Uniqueness theorem. Poisson formula. Existence of classical solutions for the homogeneous equation. Duhamel's principle and existence of classical solutions for the nonhomogeneous equation.*

Section 2. Heat conduction equation on the semi-axis.

Method of even and odd extensions of initial data. Problems with nonhomogeneous Dirichlet boundary data.

Section 3. Initial boundary value problems for the heat conduction equation.

Fourier method. Existence and uniqueness results.

Chapter 4. *Hyperbolic equations.*

Section 1. Initial value problem for the wave equation.

Classical solutions to the initial value problem for the wave equation. Characteristics. Superposition principle. Uniqueness theorem. D'Alembert formula. Kirchhoff formula. Poisson formula. Existence of classical solutions for the homogeneous equation. Duhamel's principle and existence of classical solutions for the nonhomogeneous equations in 1D, 2D, and 3D.

Section 2. Wave equation on the semi-axis.

Method of even and odd extensions of initial data. Problems with nonhomogeneous Dirichlet boundary data.

Section 3. Initial boundary value problems for the wave equation.

Fourier method. Existence and uniqueness results.

VI. Pre-taken courses

Mathematical Analysis, Functional Analysis, Ordinary Differential equations

VII. Form of the final test: examination (four-level evaluation scale)

VIII. Teaching materials and reference books

1. Mikhlin S.G., Linear equations of mathematical physics, Holt, Rinehart and Winston 1967.
2. Tikhonov A.N., Samarskii A.A., Equations of mathematical physics, Dover Publications, 2013.

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