Syllabus

I. **Course Name**: Low dimensional geometry

II. Course description and objective

The course is an introduction to the Thurston's geometries arising on 3-manifolds. Starting from surfaces and spaces of constant curvature, we come to Thurston's geometries on 3-manifolds. We focus on examples and descriptions of important class of manifolds such as toric bundles and Seifert fibrations where geometrical structures arise.

III. Master Program, 4th Term, 64 Hours, 4 Credits

IV. Course content

Chapter 1. Two-dimensional geometries.

Section 1. Topological classification of compact 2-manifolds.
Triangulation. Gluing from 2n-gon. Classification theorem.
Section 2. Spherical, hyperbolic and Euclidian geometries on compact 2-manifolds.
Metrics of constant curvature on compact 2-manifolds. Isometry groups Iso(E^2), Iso (S^2), Iso (H^2) and their properties.
Section 3. *Euler characteristic.*
Index of a generic vector field. Poincare-Hopf theorem. Gauss-Bonnet theorem.
Section 4. Fundamental group of compact 2-manifolds.
The fundamental group as a discrete co-compact subgroup of the isometry group of a universal covering. Algebraic properties of discrete subgroups and their differences for various geometries.

Chapter 2. Three dimensional manifolds of constant curvature.

Section 1. Euclidean space E^3.
The structure of isometry group Iso(E^3). Bieberbach's theorem. Space-forms.
Section 2. Spherical space S^3.
The structure of isometry group Iso(S^3).The language of quaternion. The structure of discrete subgroups of Iso(S^3). Lins spaces and other spherical space-forms.
Section 3. Hyperbolic space H^3.
Different models of hyperbolic space. The structure of isometry group Iso(H^3). The description of the isometry group in complex coordinates. Mobius group. The structure of discrete subgroups of Iso(H^3). Hyperbolic space-forms.
Section 4. Complete hyperbolic manifolds of finite volume.
Hyperbolic manifolds of finite volume obtained by gluing hyperbolic ideal simplexes. Completeness criterion of the gluing. The structure of a complete hyperbolic manifold of finite volume on a complement to the figure eight knot in S^3.

Chapter 3. Thurston geometries.

Section 1. (G,X)-Manifolds.
(G,X)-structures. The developing map. Holonomy group. Completeness criterion (G,X) - structures.
Section 2. *(G,X)-bundles.*
Connection and curvature forms of a general fiber bundle. Flat fiber bundles. Foliations and contact structures in general fiber bundles. Unit tangent bundle over surfaces.

Section 3. Eight model geometries.

Milnor classification of unimodular 3-dimensional Lee groups. Definition and classification of Thurston geometries. Examples. Thurston's conjecture.

Section 4. Thurston's geometries and special classes of 3-manifolds.

Seifert fiber spaces. Examples. Geometrization of Seifert fiber spaces. Toric bundles over the circle and their classification. Geometrization of toric bundles.

VI. Pre-taken courses

Analytic geometry, Differential geometry, General algebra.

VII. Form of the final test: examination (four-level evaluation scale)/test (two-level evaluation scale)

VIII. Teaching materials and reference books

1. W.P. Thurston, Three-dimensional Geometry and Topology: Volume 1, Princeton University Press, 1997.
2. G. Peter Scott. The geometries of 3-manifolds. Bull. London Math. Soc. 15 (1983) , no. 5, 401--487.

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