Syllabus

I. **Course Name**: Partial Differential Equations I

II. **Course description and objective**

The course includes an introduction to the Sobolev-Slobodecki spaces and time-dependent spaces. Then we consider weak solutions to linear second order elliptic partial differential equations and conditions for their regularity and weak solutions to linear evolution equations and their regularity. Finally, we study compactness method for some classes of semilinear evolution equations.

III. **Compulsory**

IV. **Master Program, 1st Term, 64 Hours, 4 Credits**

V. **Course content**

<u>Chapter 1. Sobolev-Slobodecki spaces</u>

Section 1. Integer Sobolev-Slobodecki spaces.

*Distributions. Regularizations. Spaces* $W^{k,p}(\mathrm{U})$ *and their properties. Approximation by smooth functions. Extension theorem.Trace theorem.*

Section 2. Sobolev inequalities.

*Gagliardo-Nirenberg-Sobolev inequality. Morrey's inequality. General Sobolev inequalities.*

Section 3. Compactness.

*Embedding theorems. Rellich-Kondrashov theorem. Poincaré's inequalities.*

Section 4. Fractional and negative Sobolev spaces.

*Fourier transform and its properties. Sobolev Spaces in the entire space. Equivalent norms. Fractional and negative Sobolev spaces on bounded domains.*

Section 5. Time-dependent Sobolev spaces.

*Definition of time-dependent Sobolev spaces, compactness results.*

<u>Chapter 2. Linear second order elliptic equations</u>

Section 1. Elliptic equations.

*Weak solutions. Lax-Milgram theorem.Examples.*

Section 2. Energy estimates.

*Existence of weak solutions to elliptic problems.*

Section 3. Regularity of weak solutions.

*Interior regularity. Higher interior regularity. Boundary regularity. Higher boundary regularity.*

<u>Chapter 3.</u> Generalized eigenvalue spectrum of strictly positive operators.

Section 1. Eigenvalue spectrum of symmetric operators.

*Eigenvalue spectrum of symmetric operators. Energy space of strictly positive operators. Generalized eigenvalue spectrum of strictly positive operators. Discrete spectrum of strictly positive operators with compactly embedded energy space.*

Section 2. Properties of strictly positive self-adjoint operators with discrete spectrum.

Chapter 4. Linear parabolic equations.

Section 1. Galerkin approximations.

*Weak solutions for parabolic equations. Galerkin approximations. Gronwall lemmata. Energy estimates. Existence result for weak solutions. Uniqueness of weak solutions.*

Section 2. Regularity of solutons.

*A-priory estimates. Regularity of solutions. Higher order regularity.*

Chapter 4. Linear hyperpolic equations.

Section 1. Galerkin approximations.

*Weak solutions for parabolic equations. Galerkin approximations. Energy estimates. Existence result for weak solutions. Uniqueness of weak solutions.*

Section 2. Regularity of solutons.

*A-priory estimates. Regularity of solutions. Higher order regularity.*

Chapter 5. Introduction to the theory of nonlinear partial differential equations. Compactness method.

Section 1. Berger equation.

*Weak solutions to the Berger equation. Existense and uniqueness of solutions via compactness method.*

**VI. Pre-taken courses**

Mathematical Analysis, Measure Theory and Integration, Functional Analysis

**VII. Form of the final test**: examination (four-level evaluation scale)

**VIII. Teaching materials and reference books**


1. Chueshov I., Introduction to the Theory of Infinite Dimensional Dissipative Systems, Kharkiv: Acta, 2002.
2. Evans L.C. Partial Differential Equations (2nd ed.), American Mathematical Society, Providence, Rhode Island, 2010.
3. Tartar L., An introduction to Sobolev spaces and interpolation spaces, Berlin ; New York : Springer, 2007.


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